Computing $f(A)b$, the action of a matrix function on a vector

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Abstract

Let A be a square matrix and f a function that is sufficiently smooth on the spectrum of A. Then the matrix function $f(A)$ is defined as $p(A)$, where p is the polynomial that interpolates f on the spectrum of A in the Hermite sense. Practically, it is impossible to compute $fA)$ when A is big and sparse, while it is still possible to compute $f(A)b$, b a vector.

In our talk we will address Krylov subspace techniques for computing $f(A)b$. The emphasis will be on stable restart procedures, which are mandatory even in the case that A is Hermitian, and on convergence theory. We will dedicate a large part of the talk to extensions to block methods, where one is interested in $f(A)B$, the columns of B representing several vectors b. In this contect we develop a fairly general theory for a class of block Krylov methods which comprises several different block methods considered in the literature before. We will again focus on restart procedures and convergence analysis.

This talk is based on joint work with Stefan Güttel, Kathryn Lund, Marcel Schweitzer and Daniel B. Szyld.

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