Tightening bounds on the radius of nonsingularity

David Hartman\textsuperscript{1} and Milan Hladík\textsuperscript{1}

\textsuperscript{1}Department of Applied Mathematics, Charles University, Prague, Czech Republic

Abstract

Evaluating the proximity of a given square matrix to the nearest singular one can be performed via adopting Chebyshev norm leading to so called radius of nonsingularity. Let $A$ be a matrix of a form $\mathbb{R}^{n \times n}$ and $\Delta$ is non-negative matrix of the same type, the radius of nonsingularity \cite{Poljak1997, Poljak1998} is defined by

$$d(A, \Delta) := \inf \left\{ \varepsilon > 0; \exists \text{ singular } B \forall i, j : |a_{ij} - b_{ij}| \leq \varepsilon \Delta_{ij} \right\}.$$ 

There also exists a simplified version of such radius where $\Delta$ is equal to “all ones matrix” $E$. Determining exact value of this radius even in its simplified version is known to be an NP-hard problem \cite{Poljak1998}, which leads to various lower and upper bounds \cite{Rump1997, Rump1997b}. These bounds, however, are not very tight - one of the best classical bounds has the relative error $6n$. We describe a better one based on a randomized approximation method with expected error $0.7834$ using a semidefinite relaxation \cite{Hartman2016} and discuss its possible extensions depending on various conditions given.

Keywords

Radius of non-singularity, Regularity, Interval matrix, Bounds

References

\begin{enumerate}
\end{enumerate}