Confidence Regions and Tests for Normal Models with Orthogonal Block Structure: Pivot Variables

João Tiago Mexia\(^1\), Sandra S. Ferreira\(^2\), Dário Ferreira\(^2\) and Célia Nunes\(^2\)

\(^1\)Center of Mathematics and its Applications, Faculty of Science and Technology, New University of Lisbon, Portugal

\(^2\)Department of Mathematics and Center of Mathematics, University of Beira Interior, Portugal

Abstract

Models with Orthogonal Block Structure, OBS, have variance covariance matrices that are linear combinations of pairwise orthogonal orthogonal projection matrices that add up to \(I_n\),

\[
V(\gamma) = \sum_{j=1}^{m} \gamma_j Q_j, \tag{1}
\]

see [1] and [2]. These models continue to play an important part in the theory of randomized block designs and contain the models

\[
Y = X_0\beta + \sum_{i=1}^{w} X_i Z_i, \tag{2}
\]

where \(\beta\) is fixed and the \(Z_1, \ldots, Z_w\) are independent, with null mean vectors and variance covariance matrices \(\sigma_i^2 I_{c_i}, i = 1, \ldots, w\), when the matrices \(M_i = X_i X_i^\top\) commute and \(R([X_1, \ldots, X_w]) = R^n\). We will assume normality to use pivot variables to obtain confidence regions and, through duality, test hypothesis both for:

- variance components \(\gamma_1, \ldots, \gamma_m\) and \(\sigma_1^2, \ldots, \sigma_w^2\);
- estimable functions \(\psi = c^\top \beta\) and estimable vectors \(\psi = C\beta\).

In deriving confidence regions for the \(\sigma_1^2, \ldots, \sigma_w^2\) and \(\psi\) we had to apply the Glivenko-Cantelli theorem and related results to samples of values of pivot variables. Moreover, for \(\psi\), we had to consider families of samples in order to adjust confidence ellipsoids using a technique similar to least square adjustment of linear regression.

We include a numerical application to the results of an grapevine experiment. This application is interesting in showing the good behaviour of pairs of samples for the positive and negative parts of the \(\sigma_i^2, i = 1, \ldots, w\). Then we show that we have \(\sigma_i^2 = \sigma_i^{2+} - \sigma_i^{2-}\), with \(\sigma_i^{2+}\) and \(\sigma_i^{2-}\) linear combinations of the \(\gamma_1, \ldots, \gamma_m\).
Keywords Confidence Regions, Pivot Variables, UMVUE, Variance components

Acknowledgments This work was partially supported by the Fundação para a Ciência e a Tecnologia (Portuguese Foundation for Science and Technology) through PEst-OE/MAT/UI0297/2011 (CMA), UID/MAT/00212/2013 and UID/MAT/00297/2013.

References
