Robust dynamics of real systems based on the pseudospectra

Ernest Šanca

Department of Mathematics and Informatics,
University of Novi Sad, Serbia

Abstract

The traditional approach to the stability analysis of the equilibria of real dynamical systems is based on Lyapunov stability, which consists of determining the position of the eigenvalues of the Jacobian in the complex plane. Such dynamical properties are primarily asymptotic in nature, therefore requiring usually long time scales for their realization. Additionally, potential transient behavior, which may violate the initial integrity of the system itself, when it is generating a functional response to the change it is facing, as well as the potential structural changes of the system itself, cannot be analyzed based on the expected asymptotic behavior only. The unity of rich mathematical theory, powerful theory of matrices and theory of dynamical systems offers knowledge regarding the existence of special matrix structures that are prone to high sensitivity of spectral properties, such as stability, in the presence of small perturbations. Moreover, this affectability to small perturbations in particular, is known to correspond to the effects of transient instability of otherwise asymptotically stable dynamical systems. Powerful mathematical tool that has been especially designed so as to provide better understanding of the aforementioned phenomena and adequate tool for its advanced analysis is known under the name pseudospectra. Stability indicators developed so far, exploiting the spectral properties of a matrix or the concept of GDD matrices, are simply lacking power and display drawbacks, thus providing motivation for the introduction of advancements as far as the methodology of the dynamical stability description is concerned, with the transient behavior under the functional changes in mind in the first place, highlighting the essence of pseudospectra.

This lecture aims to present concepts essentially familiar to Lyapunov stability of dynamical systems, which on one hand possess the necessary flexibility of applications in various fields of science, and the desired potential to describe complex stability aspects of real systems on the other hand. Ultimately, efficient numerical methods in determining these generalized aspects of stability, based on empirical data, so as to enable practical applications of novel concepts, shall be presented. Finally, the desired possibility of implementation and utilization of developed concepts in the multidisciplinary ambiance including physics, ecology, medicine, chemistry, engineering, economy and many more, shall be discussed.